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Short Note

Phase correction in separating *P*- and *S*-waves in elastic data

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INTRODUCTION

Two-dimensional elastic data containing reflected *P*-waves and converted *S*-waves generated by a *P*-source may be separated using dilatation and rotation calculations (Sun, 1999). The algorithm is a combination of elastic full wavefield extrapolation (Sun and McMechan, 1986; Chang and McMechan, 1987, 1994) and wave-type separation using dilatation (divergence) and rotation (curl) calculations (Dellinger and Etgen, 1990). It includes (1) downward extrapolating the (multicomponent) elastic data in an elastic velocity model using the elastic wave equation, (2) calculating the dilatation to represent pure *P*-waves and calculating the rotation to represent pure *S*-waves at some depth, and (3) upward extrapolating the dilatation in a *P*-velocity model and upward extrapolating the rotation in an *S*-velocity model, using the acoustic wave equation for each.

Assume that the parameters in elastic seismic data are displacement components. Calculating the dilatation and rotation involves spatial derivatives on displacement components that induce a $\pi/2$ phase shift. Thus the separated *P*- and *S*-waves using dilatation and rotation calculations will have wavelets that are phase shifted by $\pi/2$ compared to the wavelets in the elastic data. To illustrate this phase shift, if a wavelet in the elastic data is the function (Figure 1a)

$$f(t) = -[(t - t_0)/a]\exp[-(t - t_0)^2/a], \qquad (1)$$

where t is time and t_0 and a are constants, then the wavelets in the separated *P*- and *S*-waves will be a $\pi/2$-phase-shifted function (Figure 1b). Removing this phase shift is necessary to restore the original wavelet.

The method described in this paper corrects the phase in the separated *P*- and *S*-waves. A compensating $-\pi/2$ phase shift by a Hilbert transform will achieve this result. Although the $\pi/2$ phase shift is induced in the derivatives with respect to spatial variables, it is not necessary to perform the phase correction in the spatial domain. We have the freedom to perform the phase correction either in the spatial domain or in the time domain. In this paper, it is performed in the time domain.

This paper treats two-dimensional seismic data. Rectangular (x, z) space coordinates are used for illustration. Both elastic and acoustic wave equations (Appendix A) are used for wavefield extrapolations. Finite difference simulations, of fourth-order accuracy in spatial derivatives and second-order accuracy in time derivatives (Dablain, 1986), of the elastic wave equation (A-1) and of the acoustic wave equation (A-2) are used. The grid spacing $h = 0.01$ km in both x- and z-coordinates and the time sampling increment $\Delta t = 0.001$ s are used throughout this paper.

THEORY

This section contains a mathematical derivation of the origin of the $\pi/2$ phase shift induced in separating the *P*- and *S*-waves, and is followed by the approach to correct this phase shift.

Let $\mathbf{U_P}$ and $\mathbf{U_S}$ denote the displacement vectors of the propagating *P*- and *S*-waves, respectively. In two-dimensional elastic downward extrapolation, the wavefield (displacement vector) $\mathbf{U}$ can be expressed as a linear combination of $\mathbf{U_P}$ and $\mathbf{U_S}$:

$$\mathbf{U} = \phi_0\mathbf{U_P} + \theta_0\mathbf{U_S}, \qquad (2)$$

where ϕ_0 and θ_0 are constants. Lamé's theorem (Aki and Richards, 1980) states that $\mathbf{U_P}$ and $\mathbf{U_S}$ can be derived as the gradient of a scalar compressional potential A and the curl of a vector shear potential $\mathbf{B}$, respectively:

$$\mathbf{U_P} = \nabla A, \qquad (3a)$$

$$\mathbf{U_S} = \nabla \times \mathbf{B}. \qquad (3b)$$

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For a harmonic plane wave, the compressional potential A can be expressed as a scalar function

$$A = -ie^{i(k_{px}x + k_{pz}z - \omega t)}, \quad (4a)$$

where k_{px} and k_{pz} denote the P-wave spatial wavenumbers in the x- and z-directions, respectively, ω denotes the angular frequency, and $i = \sqrt{-1}$. The shear potential **B** can be expressed as a vector function

$$\mathbf{B} = -ie^{i(k_{sx}x + k_{sz}z - \omega t)}\mathbf{a}_y, \quad (4b)$$

where k_{sx} and k_{sz} denote the S-wave spatial wavenumbers in the x- and z-direction, respectively, and the unit vectors in x-, y- and z-directions are denoted by $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$, respectively. Then the displacement vectors $\mathbf{U_P}$ and $\mathbf{U_S}$ can be derived by substituting equations (4a) and (4b) into equations (3a) and (3b), respectively. The results are

$$\mathbf{U_P} = (k_{px}\mathbf{a}_x + k_{pz}\mathbf{a}_z)\, e^{i(k_{px}x + k_{pz}z - \omega t)} \quad (5a)$$

and

$$\mathbf{U_S} = (k_{sz}\mathbf{a}_x - k_{sx}\mathbf{a}_z)\, e^{i(k_{sx}x + k_{sz}z - \omega t)}. \quad (5b)$$

Substituting equations (5a) and (5b) into equation (2) gives the expression for the displacement vector **U**:

$$\mathbf{U} = \left[\phi_0(k_{px}\mathbf{a}_x + k_{pz}\mathbf{a}_z)\, e^{i(k_{px}x + k_{pz}z)} + \theta_0(k_{sz}\mathbf{a}_x - k_{sx}\mathbf{a}_z)\, e^{i(k_{sx}x + k_{sz}z)}\right]e^{-i\omega t}. \quad (6)$$

As spherical waves can be expressed as combinations of plane waves, downward propagating wavefields can be represented by equation (6).

For separating P- and S-waves, the dilatation ϕ is calculated to represent the P-waves and the (vector) rotation $\theta\mathbf{a}_y$ is calculated to represent the S-waves by substituting equation (6) into equations (3a) and (3b):

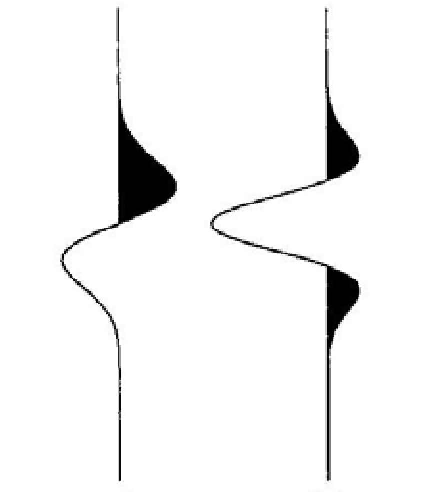

FIG. 1. A wavelet (a) and its $\pi/2$ phase shift (b). The amplitudes of both wavelets have been normalized.

$$\phi(x, z, t) = \nabla \cdot \mathbf{U} = \phi_0 \nabla \cdot \mathbf{U_P} = i\phi_0\left(k_{px}^2 + k_{pz}^2\right) e^{i(k_{px}x + k_{pz}z - \omega t)}, \quad (7a)$$

$$\theta(x, z, t)\mathbf{a}_y = \nabla \times \mathbf{U} = \theta_0 \nabla \times \mathbf{U_S} = i\theta_0\left(k_{sx}^2 + k_{sz}^2\right) e^{i(k_{sx}x + k_{sz}z - \omega t)}\mathbf{a}_y. \quad (7b)$$

The divergence of the S-wave displacement ($\nabla \cdot \mathbf{U_S}$) and the curl of the P-wave displacement ($\nabla \times \mathbf{U_P}$) vanish identically in equations (7a) and (7b).

Equations (5a) and (7a) show that the dilatation ϕ differs from the P-wave displacement $\mathbf{U_P}$ in that the harmonic term $\exp[i(k_{px}x + k_{pz}z - \omega t)]$ in equation (7a) is multiplied by a factor i, whereas that in equation (5a) is not. The factor i, which is $\exp(i\pi/2)$, causes the $\pi/2$ phase shift. Similarly, equations (5b) and (7b) show that a $\pi/2$ phase shift also appears in the separated S-waves.

The $\pi/2$ phase shift can be corrected by imposing a compensating $-\pi/2$ phase shift to both ϕ and θ in equations (7a) and (7b). A Hilbert transform will achieve this purpose. Since the factor i in equations (7a) and (7b) multiplies the entire harmonic term $\exp[i(k_x x + k_z z - \omega t)]$ rather than the spatial variables x or z, we have the freedom to perform the $-\pi/2$ phase shift in the time domain. Imposing the $-\pi/2$ phase shift can be achieved as long as the factor i is removed.

For separating the P- and S-waves, dilatation ϕ and rotation θ are computed at some depth z_1 and can be represented by functions of x and t (with constant z):

$$\begin{bmatrix}\phi\\ \theta\end{bmatrix} = \begin{bmatrix}\phi(x, t)\\ \theta(x, t)\end{bmatrix}. \quad (8)$$

The Hilbert transform with respect to t (Claerbout, 1976, 20–23) will be applied on equation (8) to correct the phase shift.

A SYNTHETIC EXAMPLE

Figure 2 shows an isotropic, two-dimensional laterally inhomogeneous elastic model of 4.0 km horizontal extent and 2.2 km depth extent, with one dipping reflector and one horizontal reflector. The synthetic common-source elastic data generated for this model with P-source at $(x, z) = (2.0, 0.14)$ km are presented in Figure 3. The data are generated with absorbing

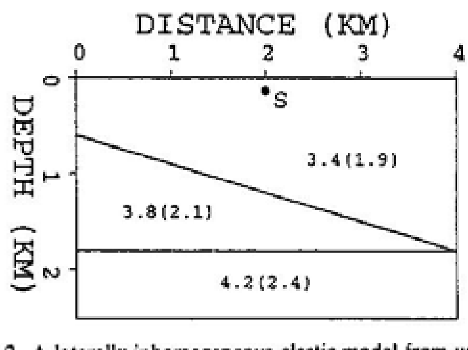

FIG. 2. A laterally inhomogeneous elastic model from which the synthetic elastic data in Figure 3 are generated. The numbers outside the parentheses denote P-velocities (in km/s), and the numbers inside the parentheses denote S-velocities (in km/s). S is the source location.

boundaries on all (top, bottom, left, and right) edges. The direct arrival has been muted, so the data include only the reflected P-waves and the converted S-waves.

The elastic data (Figure 3) are downward extrapolated into a homogeneous elastic velocity model using elastic wave equation (A-1). During downward extrapolation, at some depth z_1 (here, $z_1 = 0.2$ km) the dilatation $\phi(x, t)$ is computed to extract the P-waves (Figure 4a), and the rotation $\theta(x, t)$ is computed to extract the S-waves (Figure 4b). The wavelets of the input elastic data (Figure 3) are shaped as in Figure 1a, but the wavelets of the separated P- and S-waves (Figures 4a, b) are shaped as in Figure 1b, i.e., phase shifted by $\pi/2$.

The Hilbert transform is applied to each trace of the dilatation and the rotation (Figures 4a, b), and the results are presented in Figures 5a and 5b, respectively. The wavelets in Figures 5a and 5b are again shaped as those in Figures 3a and 3b, implying that the phase has been corrected.

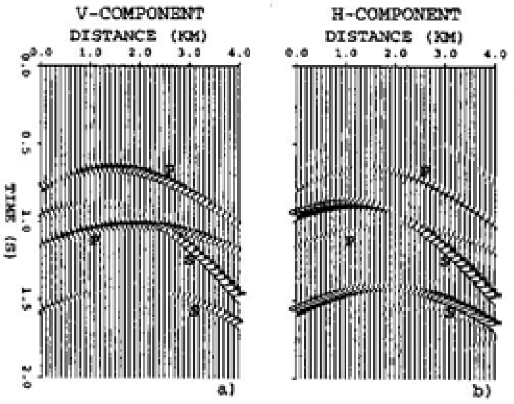

FIG. 3. Synthetic common-source elastic data generated from the model in Figure 2: (a) vertical component, (b) horizontal component. P = P-wave; S = S-wave.

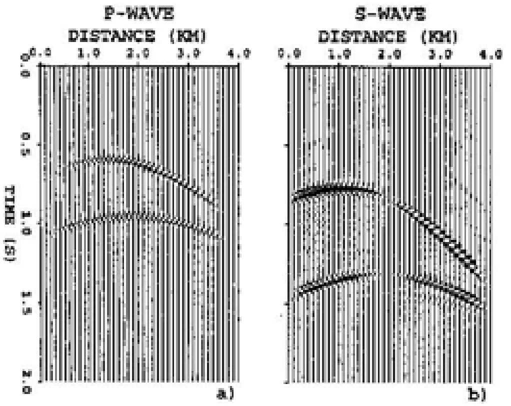

FIG. 4. Separated P-waves (a) and S-waves (b) extracted at depth 0.2 km.

Figures 5a and 5b are the separated waves extracted at depth z_1, not at the earth's surface. To reconstruct the separated waves at the earth's surface, Figure 5a needs to be upward extrapolated in a P-velocity model and Figure 5b needs to be upward extrapolated in an S-velocity model, both using acoustic wave equation (A-2). The procedure has been detailed by Sun (1999) and is not repeated here.

DISCUSSION AND CONCLUSIONS

The algorithm to separate P- and S-waves was previously developed. That algorithm performs separation of elastic data into pure P- and S-waves. The $\pi/2$ phase shift in the wavelets is unavoidable because spatial derivatives are required operations in separating the P- and S-waves. This paper shows that the phase of the wavelets can be corrected by applying a Hilbert transform with respect to time after the P- and S-waves have been separated. The $\pi/2$ phase shift is induced in the derivatives with respect to x and z. It is difficult to correct the phase in the x-z domain, but easily implemented via a Hilbert transform in the t domain.

In real seismic data, the parameter may be particle velocity, which is the time derivative of displacement. With real data, we should simply substitute the displacement, dilatation and rotation in equations (2)–(8) by their individual time derivatives. The phase correction algorithm will remain valid.

Correcting the phase of the wavelets can also be achieved using deconvolution and wave shaping (Yilmaz, 1987, 82–153). The purpose of this paper is to demonstrate that the phase shift in separating P- and S-waves is exactly $\pi/2$ and, therefore, a compensating $-\pi/2$ phase shift can be one of the possible phase correcting approaches, and it is exact.

This paper assumes that the source wavelet is antisymmetric (Figure 1a). The $\pi/2$-phase-shifted wavelets of the separated P- and S-waves are then zero phase. However, if the phase-shifted wavelets are preferred, the phase correction procedure in this paper can be omitted.

As the separated waves are functions of x and t, ideally we should be able to perform the phase correction by applying a Hilbert transform with respect to either x or t. A singularity happens at $k_x = 0$ (a result of multiplying $-i$ in the $k_x < 0$ region

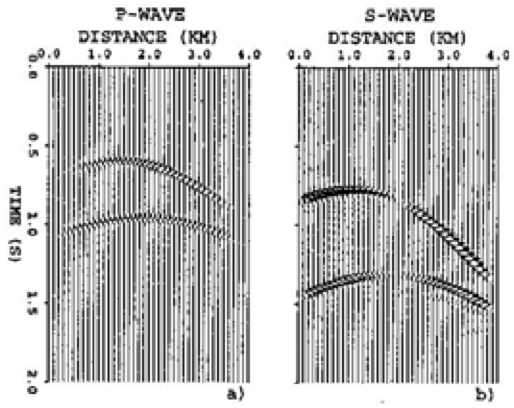

FIG. 5. The phase-corrected versions of the separated P-waves (a) and S-waves (b) extracted at depth 0.2 km.

and multiplying i in the $k_x > 0$ region) if a Hilbert transform with respect to x is performed. Similarly, a singularity happens at $\omega = 0$ if a Hilbert transform with respect to t is performed. As the separated waves may have very high apparent velocities with nonzero energy in the vicinity of $k_x = 0$, a Hilbert transform with respect to x is not recommended since the singularity at $k_x = 0$ may cause nasty artifacts (Dellinger, 1991). On the other hand, since the separated waves are usually band limited with little energy in the vicinity of $\omega = 0$, a Hilbert transform with respect to t is recommended since the artifacts caused by the singularity at $\omega = 0$ are negligible.

In this paper, the dilatation and rotation calculations in equations (7a) and (7b) are performed using finite differences in the spatial domain. They can also be performed in the wavenumber domain using Fourier transforms with respect to x and z (Dellinger and Etgen, 1990). The subsequent phase correction procedure will be the same as described above.

This paper only treats two-dimensional seismic data in isotropic media. The Hilbert transform for the phase correction is with respect to t only, so it will remain the same if we are to treat three-dimensional seismic data. Therefore, the extension of this phase correction algorithm to three dimensions will be straightforward. In anisotropic media, since the wave separation will still be performed by the dilatation and rotation calculations in equations (7a) and (7b), the term $\exp(-i\omega t)$ and also the phase shift problem remain the same as those in isotropic media. The phase correction algorithm in this paper will be equally effective.

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APPENDIX A

TWO-DIMENTIONAL WAVE EQUATIONS

Isotropic media are assumed in this paper, and both elastic and acoustic wave equations are used. The two-coupled wave equations for two-dimensional elastic extrapolation are (Kelly et al., 1976; Sun and McMechan, 1986)

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2} + (\alpha^2 - \beta^2)\frac{\partial^2 w}{\partial x \partial z} + \beta^2 \frac{\partial^2 u}{\partial z^2}$$

and

$$\frac{\partial^2 w}{\partial t^2} = \alpha^2 \frac{\partial^2 w}{\partial z^2} + (\alpha^2 - \beta^2)\frac{\partial^2 u}{\partial x \partial z} + \beta^2 \frac{\partial^2 w}{\partial x^2}, \quad \text{(A-1)}$$

where t is time, $u = u(x, z, t)$ and $w = w(x, z, t)$ are the horizontal and vertical displacement components, respectively, and α and β are the P- and S-velocities, respectively. The acoustic wave equation is

$$\frac{\partial^2 Q}{\partial t^2} = c^2\left(\frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial z^2}\right), \quad \text{(A-2)}$$

where $Q = Q(x, z, t)$ is the wave function and c is the acoustic velocity. In both equation (A-1) and equation (A-2), the x-axis (horizontal position) points to the right and the z-axis (depth) points downward.